Transient Behaviour of Large Transformer Windings Taking Capacitances and Eddy Currents into Account

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Transformer windings, as oscillatory systems, respond to impulse excitation by starting to oscillate. In the paper, a detailed transient circuit analysis of an auto-transformer responding to a standard lightning impulse (LI) has been performed, taking into account each single turn of the two windings. The corresponding inductance and capacitance matrices used in the circuit analysis are obtained by FEM-calculations. To get the resistances of the turns, eddy currents have to considered. Since the turns (continuous transposed conductors, CTC's) consist of a certain number of single strands to keep the eddy current losses low, an appropriate eddy current calculation by FEM has to be performed, considering each single strand of the turns. In the present example, the resulting resistances of the turns are typically several hundred times higher than the respective DC resistances. This has a significant influence on the damping behaviour of the high frequency oscillations of the system.

***Index Terms*— Continuous transposed conductors, eddy currents, transient behavior, transformer windings**

I. Introduction

The study of very fast transient (VFT) phenomena in transformer windings is important in the design of large transformers for various reasons [1]-[2]. In order to get a sufficiently high frequency resolution, each single turn (the turns are assumed to be electrically short, so there are no distributed parameters along the turns) of the transformer has to be modelled by FEM to get the inductance and capacitance matrices for a detailed circuit analysis [1]. The scheme of the analyzed auto-transformer with 2x360 turns (each turn is a 19-strand CTC) is shown in Fig. 1. A standard lightning impulse (1.2 μs/ 50 μs) is applied at the high voltage front end (turn 361, center of winding two, see Fig. 1). The inner winding is grounded at each end. The diameters are: core: 994 mm; inner winding: 1102/1258 mm; outer winding: 1468/1634 mm. The height of the windings is 1200 mm and the distances of the windings to the yokes are 150 mm. The modeled structure can be seen on the left of Fig. 3. The cross–section of a single strand (copper) is 1.2 mm x 7.5 mm. The arrangement (slightly simplified) of the 19 strands within a turn is shown in the right of Fig. 3.

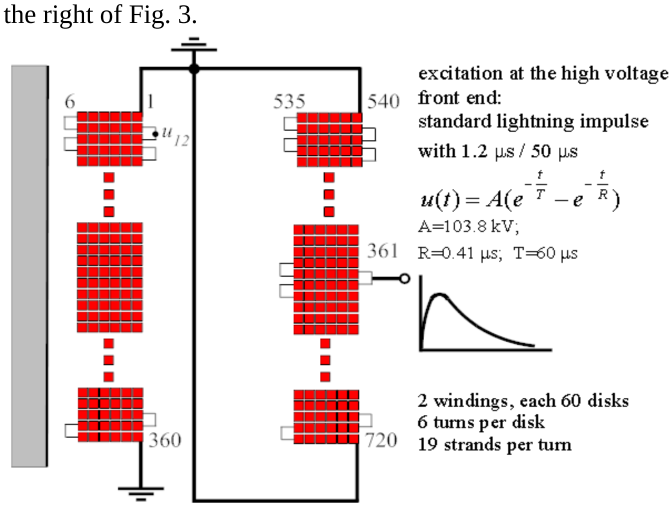

Fig. 1. Scheme of the analyzed auto-transformer consisting of two windings with 2x60 disks and 2x6x60 turns with a total number of 13680 single strands.

II. Circuit Model

In the circuit model for the auto-transformer, the turn currents and voltages are used as state variables as can be seen in Fig. 2 for a similar problem consisting of three turns only. Equation (1) describes the appropriate ordinary differential equations system for this small problem. The full model comprises 720 turns and therefore 1440 state variables. The numbering of the turns in Fig. 1 is according to the numbering in the circuit model. The equations system is solved by backward Euler method. The resistances are updated during every time step with actual values obtained by a finite element eddy current calculation, in an iterative way. The convergence of the iterative process is fast (at least for this problem), usually only one iteration step is necessary for each time step.Although the simple Euler method is used, 'numerical damping effects' can be neglected due to the fine time steps.

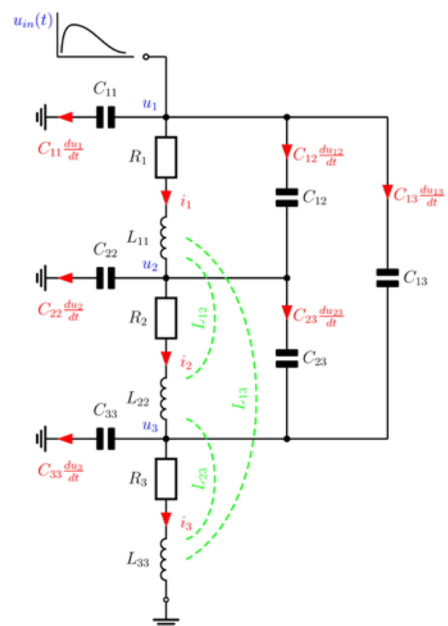

Fig. 2. Circuit model for 3 turns. All branches and nodes are coupled via complete capacitance and inductance matrices.

$$
\begin{bmatrix}
L_{11} & L_{12} & L_{13} & 0 & 0 \\
L_{12} & L_{22} & L_{13} & 0 & 0 \\
L_{13} & L_{23} & L_{33} & 0 & 0 \\
0 & 0 & 0 & C_{12}+C_{22}+C_{23} & -C_{23} \\
0 & 0 & 0 & -C_{23} & C_{13}+C_{23}+C_{33}
\end{bmatrix}
\begin{Bmatrix}
\frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{di_3}{dt} \\ \frac{du_2}{dt} \\ \frac{du_3}{dt}
\end{Bmatrix} +
$$

$$
+\begin{bmatrix}
R_1 & 0 & 0 & 1 & 0 \\
0 & R_2 & 0 & -1 & 1 \\
0 & 0 & R_3 & 0 & -1 \\
-1 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0
\end{bmatrix}
\begin{Bmatrix}
i_1 \\ i_3 \\ i_3 \\ u_2 \\ u_3
\end{Bmatrix} =
\begin{Bmatrix}
u_1 \\ 0 \\ 0 \\ C_{12}\frac{du_1}{dt} \\ C_{13}\frac{du_1}{dt}
\end{Bmatrix}.
$$

(1)

III. Electromagnetic Analysis

The finite element method is applied to the axisymmetric eddy current problem with N_L single strands, whereby the current condition is directly implemented in the multipath formulation [2]. The governing equations are given by:

$curl\mathbf{H} - \mathbf{J} = 0$; $\mathbf{B} = curl\mathbf{A}$

$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} + \frac{\dot{u}}{l}\mathbf{e}_I$; $\dot{u}$ ... source voltage per turn

$u = \sum_{m=1}^{N_L} u_m N_{\Omega_{c_m}}$ ... time integral of the source voltage

$curl\nu curl\mathbf{A} + \sigma\frac{\partial \mathbf{A}}{\partial t} - \sigma\dot{u}\mathbf{e}_\varphi = 0$; $\mathbf{e}_\varphi$... unit vector in current direction

$\mathbf{A} = A\mathbf{e}_\varphi$, $\mathbf{A} = \sum_{k\in N}\mathbf{N}_k$; $\mathbf{N}_k = N_k\mathbf{e}_\varphi$; $N_{\Omega_{c_m}} = \sum_{k\in\Omega_{c_m}} N_k = \begin{cases} 1 & \text{in } \Omega_{c_m} \\ 0 & \text{elsewhere} \end{cases}$.

Galerkin equations:

$$
\begin{aligned}
&\sum_{k\in N} A_k \int_\Omega curl\mathbf{N}_i \cdot \nu\, curl\mathbf{N}_k d\Omega + \sum_{k\in N}\dot{A}_k \int_\Omega \sigma\mathbf{N}_i\cdot\mathbf{N}_k d\Omega - \\
&-\sum_{m=1}^{N_L}\dot{u}_m \int_{\Omega_{c_m}} \sigma\mathbf{N}_i\cdot\mathbf{N}_{\Omega_{c_m}} d\Omega = 0 \qquad i\in N \\
&-\sum_{k\in N}\dot{A}_k \int_{\Omega_{c_l}} \sigma\mathbf{N}_k\cdot\mathbf{N}_{\Omega_{c_l}} d\Omega + \sum_{m=1}^{N_L}\dot{u}_m \int_{\Omega_{c_l}} \sigma\mathbf{N}_{\Omega_{c_m}}\cdot\mathbf{N}_{\Omega_{c_l}} d\Omega = i_l \\
&\qquad\qquad\qquad\qquad l = 1,2,3,\ldots,N_L
\end{aligned}
\quad . \qquad (2)
$$

The currents i_l in the strands are taken from the branch currents of the circuit description, divided by 19. The solution of equation (2) is used to update the resistances (the parallel connection of the respective 19 strand resistances for each turn) in the circuit model in an iterative way.

IV. Numerical Results

The left plot in Fig. 3 shows the potential distribution at a time of 10 μs (the maximum value of the exciting LI is 100 kV at t=2 μs). On the right in Fig. 3, the eddy current distribution in turn 528 can be seen at 10 μs for all 19 strands. At earlier time instances, the current distribution is even more extreme.

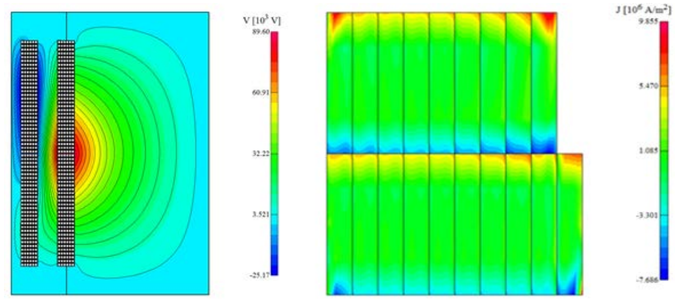

Fig. 3. Potential distribution and eddy current distribution (turn 528, 19 strands, geometry slightly simplified)) at t=10 μs.

In Fig. 4, the time dependence of the potential of turn 12 can be seen for different values of time steps. The position of the turn is indicated in Fig. 1. As reference solution, the results obtained by a lumped parameter model is used. In this lumped model, the inductances and capacitances of all 12 turns of each two neighboring disks are lumped to one value. Therefore, the reference value is the first non-zero value in the lumped model. The values for the L- and C-matrices are derived from analytical approximation methods (for C) and series expansions (for L), although there is the possibility of using the FEM results for lumping. The differences between the reference curve and the blue curve (where the higher frequencies are already suppressed due the large time step) are obviously due to the inaccurate L- and C-values. At finer time steps, higher frequencies up to 8 MHz can be seen. This is also approximately the limit of the single-turn method.

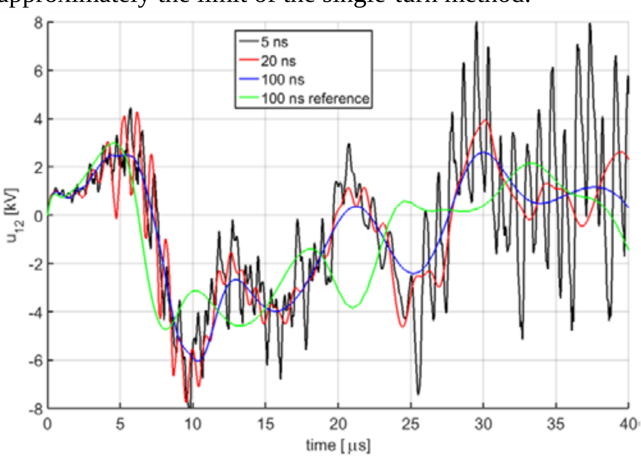

Fig. 4. Time dependency of u_{12} for 3 different time steps (5 ns, 20 ns and 100 ns). The reference solution (green) is taken from a lumped model.

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